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# CONTROL OF SLOWLY VARYING LPV SYSTEMS: AN APPLICATION TO FLIGHT CONTROL *

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## Abstract

Recent results in parameter-dependent control of linear parameter-varying (LPV) systems are applied to the problem of designing gain-scheduled pitch rate controllers for the F-16 VISTA (Variable-Stability In-Flight Simulator Test Aircraft). These methods, based on parameter-dependent quadratic Lyapunov functions, take advantage of known *a priori* bounds on the parameters' rates of variation (the bounds may themselves be parameter-varying). The controller achieves an induced-$\mathcal{L}_2$-norm performance objective; Level 1 flying qualities are predicted. Sub-optimal solutions are obtained by solving a convex optimization problem described by linear matrix inequalities (LMIs). Incorporation of $D$-$K$ iteration with "constant $D$-scales" provides robustness to time-varying uncertainty. Parameter-varying performance weights are used to shape the desired performance at different points in the design envelope.

## 1 Introduction

The area of analysis and control of *linear parameter-varying* (LPV) systems has received much recent attention, primarily in order to develop systematic techniques for gain-scheduling. These systems resemble linear systems that depend on one or more time-varying parameters; nonlinear systems are often modelled in this form via a parameterized family of linearizations. The analysis of LPV systems differ from that of linear *time-varying* (LTV) systems in that it considers whole *families* of parameter trajectories; moreover, the parameter values are available only in real time, not in advance.

The classical approach to gain-scheduled $\mathcal{H}_\infty$ control involves designing an (LTI) $\mathcal{H}_\infty$ controller for each of a parameterized family of linearizations and then interpolating controller gains by operating condition. This heuristic approach yields satisfactory results if the parameters are sufficiently "slowly-varying."[14] Early results in so-called "LPV synthesis" explicitly account for this time-variation using scaled small-gain arguments[3,12] or single quadratic Lyapunov functions (SQLF);[4,6,8] those designs tend to be conservative, though, partly because they allow the parameters to vary arbitrarily quickly. More recent results[2,7,16,17] use parameter-dependent Lyapunov functions (PDLF) to factor in *a priori* bounds on the parameters' rates of variation, reducing this conservatism.

In this paper the PDLF technique[7,16,17] is incorporated into the $D$-$K$ iteration framework in order to design robust, parameter-varying pitch rate controllers for the F-16 VISTA (Variable Stability In-Flight Simulator Test Aircraft). Sub-optimal solutions are obtained by solving a convex optimization problem described by a system of linear matrix inequalities (LMIs), for which efficient algorithms are available. Parameter-varying performance weights are used to smoothly vary the desired performance within the design envelope.

The remainder of this paper is organized as follows. Section 2 reviews some results on the control of LPV systems. Section 3 provides a robustness test for LPV systems. Section 4 applies these techniques to the F-16 VISTA. Section 5 summarizes the paper and discusses future prospects.

This paper uses standard notation. In addition, $\mathcal{S}^{n\times n}$ denotes the set of real, symmetric, $n \times n$ matrices. For any matrix $X \in \mathcal{S}^{n\times n}$, $X > 0$ and $X < 0$ respectively denote positive-definiteness (all of its eigenvalues are positive) and negative-definiteness



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(all of its eigenvalues are negative). Continuously differentiable functions are called $\mathcal{C}^1$, and $\mathcal{C}^1(V, W)$ denotes the set of $\mathcal{C}^1$ functions from $V$ to $W$.

# 2 Control of LPV Systems

This section reviews some results on the control of continuous-time, linear parameter-varying systems. The reader may consult the applicable references[7,16,17] (and the papers cited therein) for details.

An LPV system $G(s, \rho)$ is a finite-dimensional linear system

$$\begin{bmatrix} \dot{x}(t) \\ e(t) \end{bmatrix} = \begin{bmatrix} A(\rho(t)) & B(\rho(t)) \\ C(\rho(t)) & D(\rho(t)) \end{bmatrix} \begin{bmatrix} x(t) \\ d(t) \end{bmatrix} \quad (2.1)$$

whose state-space data are known continuous functions of time-varying parameters denoted by $\rho \in \mathbf{R}^s$. The $s$ parameter values are not known in advance; rather, they are measured in real-time. Assume that the bounded vector-valued parameter signal $\rho \in \mathcal{L}_\infty$ is a piecewise-$\mathcal{C}^1$ function of time and that there exists a compact set $\mathcal{P} \subset \mathbf{R}^s$ for which $\rho(t) \in \mathcal{P}$ for all $t \geq 0$.

Also assume that the rates of variation of the first $\bar{s}$ parameters $\rho_1, \ldots, \rho_{\bar{s}}$ are each bounded in magnitude by known positive scalars $\nu_1, \ldots, \nu_{\bar{s}}$, i.e.,

$$|\dot{\rho}_i(t)| \leq \nu_i \text{ for all } t > 0 \ (i = 1, \ldots, \bar{s}).$$

Denote these $\bar{s}$ rate-limited parameters and rate bounds by $\bar{\rho} := (\rho_1, \ldots, \rho_{\bar{s}}) \in \mathbf{R}^{\bar{s}}$ and $\nu := (\nu_1, \ldots, \nu_{\bar{s}}) \in \mathbf{R}^{\bar{s}}$, respectively. Now $\nu$ may itself be a continuous function of the parameters $\rho$, so that parameter trajectories must obey the differential inclusion

$$|\dot{\bar{\rho}}_i(t)| \leq \nu_i(\rho(t)) \text{ for all } t > 0 \ (i = 1, \ldots, \bar{s})$$

Parameter trajectories satisfying the above conditions for given $\mathcal{P}$ and $\nu$ will be called *allowable.* Note that, at the cost of added notation, one can expand the results in this section to separate upper and lower bounds of the rates of variation.

## 2.1 Induced $\mathcal{L}_2$-norm Analysis

Given a family of allowable parameter trajectories defined by a parameter set $\mathcal{P}$ and a rate-of-variation bound $\nu$, one can bound the induced $\mathcal{L}_2$ norm of an LPV system using a parameter-dependent quadratic Lyapunov function.

**Lemma 2.2** Given the LPV system in (2.1) and a performance level $\gamma > 0$, suppose there exists a matrix function $W \in \mathcal{C}^1(\mathbf{R}^{\bar{s}}, \mathcal{S}^{n \times n})$ such that $W(\bar{\rho}) > 0$ and (omitting dependence on $\rho$ and $\bar{\rho}$)

$$\begin{bmatrix} A^T W + WA + \sum_{i=1}^{\bar{s}} \beta_i \frac{\partial W}{\partial \bar{\rho}_i} & WB & C^T \\ B^T W & -\gamma I & D^T \\ C & D & -\gamma I \end{bmatrix} < 0 \quad (2.3)$$

for all $\beta_i \in [-\nu_i(\rho), \nu_i(\rho)]$ at each parameter value $\rho \in \mathcal{P}$. Then for any allowable parameter trajectory the LTV system governed by (2.1) is exponentially stable. Furthermore, there exists $\gamma_1 \in [0, \gamma)$ for which $\|e\|_2 \leq \gamma_1 \|d\|_2$ for all $d \in \mathcal{L}_2$ (if $x(0) = 0$).

The constraints $W(\bar{\rho}) > 0$ and (2.3) represent convex *linear matrix inequality* (LMI) constraints on the variables $W$ and $\gamma$. Although these LMIs are clearly infinite dimensional, one can compute solutions using the approximate method described in the sequel.

## 2.2 Output-feedback synthesis

Consider an LPV plant in the standard form

$$\begin{bmatrix} \dot{x} \\ e \\ y \end{bmatrix} = \begin{bmatrix} A(\rho) & B_1(\rho) & B_2(\rho) \\ C_1(\rho) & D_{11}(\rho) & D_{12}(\rho) \\ C_2(\rho) & D_{21}(\rho) & D_{22}(\rho) \end{bmatrix} \begin{bmatrix} x \\ d \\ u \end{bmatrix} \quad (2.4)$$

where $x \in \mathbf{R}^n$, $e \in \mathbf{R}^{n_e}$, $d \in \mathbf{R}^{n_d}$, $u \in \mathbf{R}^{n_u}$, and $y \in \mathbf{R}^{n_y}$, and other quantities are dimensioned appropriately. By assuming *regularity* (i.e., $D_{12}$ and $D_{21}$ are full rank) and $D_{11} = 0$, (2.4) can be transformed into[6]

$$\begin{bmatrix} \dot{x} \\ e_1 \\ e_2 \\ y \end{bmatrix} = \begin{bmatrix} A(\rho) & B_{11}(\rho) & B_{12}(\rho) & B_2(\rho) \\ C_{11}(\rho) & 0 & 0 & 0 \\ C_{12}(\rho) & 0 & 0 & I \\ C_2(\rho) & 0 & I & 0 \end{bmatrix} \begin{bmatrix} x \\ d_1 \\ d_2 \\ u \end{bmatrix} \quad (2.5)$$

The *LPV $\gamma$-Performance/$\nu$-Variation problem* consists of finding a parameter-varying controller

$$\begin{bmatrix} \dot{x}_c \\ u \end{bmatrix} = \begin{bmatrix} A_k(\rho, \dot{\bar{\rho}}) & B_k(\rho, \dot{\bar{\rho}}) \\ C_k(\rho, \dot{\bar{\rho}}) & D_k(\rho, \dot{\bar{\rho}}) \end{bmatrix} \begin{bmatrix} x_c \\ y \end{bmatrix}$$

(possibly dependent on $\dot{\bar{\rho}}$) for which the closed-loop system (omitting dependence on $\rho$ and $\dot{\bar{\rho}}$)

$$\begin{bmatrix} \dot{x} \\ \dot{x}_c \\ \hline e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} A_{clp} & B_{clp} \\ C_{clp} & D_{clp} \end{bmatrix} \begin{bmatrix} x \\ x_c \\ \hline d_1 \\ d_2 \end{bmatrix}$$

satisfies the conditions of Lemma 2.2 for a desired closed-loop norm $\gamma > 0$. The following theorem gives necessary and sufficient conditions for solvability.

**Theorem 2.6** Given the compact set $\mathcal{P}$, the vector-valued function $\nu(\rho)$, the scalar $\gamma > 0$, and the open-loop system (2.5), the LPV $\gamma$-Performance/$\nu$-Variation problem is solvable if and only if there exist matrix functions $X \in \mathcal{C}^1(\mathbf{R}^{\bar{s}}, \mathcal{S}^{n\times n})$ and $Y \in \mathcal{C}^1(\mathbf{R}^{\bar{s}}, \mathcal{S}^{n\times n})$ such that (omitting dependence on $\rho$ and $\bar{\rho}$)

$$\begin{bmatrix} \begin{array}{c}\tilde{A}^T X + X\tilde{A} - \gamma C_2^T C_2 \\ +\sum_{i=1}^{\bar{s}} \pm\nu_i \frac{\partial X}{\partial \bar{\rho}_i}\end{array} & XB_{11} & C_1^T \\ B_{11}^T X & -\gamma I & 0 \\ C_1 & 0 & -\gamma I \end{bmatrix} < 0 \tag{2.7}$$

$$\begin{bmatrix} \begin{array}{c}\hat{A}Y + Y\hat{A}^T - \gamma B_2 B_2^T \\ -\sum_{i=1}^{\bar{s}} \pm\nu_i \frac{\partial Y}{\partial \bar{\rho}_i}\end{array} & YC_{11}^T & B_1 \\ C_{11} Y & -\gamma I & 0 \\ B_1^T & 0 & -\gamma I \end{bmatrix} < 0 \tag{2.8}$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0 \tag{2.9}$$

for all $\rho \in \mathcal{P}$, where

$$\begin{array}{ll} \tilde{A} = A - B_{12}C_2, & C_1^T = [C_{11}^T \; C_{12}^T] \\ \hat{A} = A - B_2 C_{12}, & B_1 = [B_{11} \; B_{12}] \end{array}$$

The notation $\pm(\cdot)_i$ indicates that the inequalities must hold for every combination of $+(\cdot)_i$ and $-(\cdot)_i$. Therefore, (2.7) and (2.8) each represent $2^{\bar{s}}$ LMI's.

Controllers specified by arbitrary parameter-varying solutions $X(\bar{\rho})$ and $Y(\bar{\rho})$ typically depend explicitly on the parameter derivative $\dot{\bar{\rho}}$. On the other hand, Becker[7] shows that in the case where the conditions of Theorem 2.6 are satisfied when $X$ or $Y$ is *constant* with respect to $\bar{\rho}$, one can derive formulas for strictly proper ($D_k = 0$) controllers that are independent of $\dot{\bar{\rho}}$. Specifically,

$$\begin{aligned} A_k &= N^{-1}[A^T + X(A + B_2F + LC_2)Y + Z]M^{-T} \\ B_k &= N^{-1}XL \\ C_k &= FYM^{-T} \end{aligned}$$

where

$$\begin{aligned} F &= -[\gamma B_2^T Y^{-1} + C_{12}] \\ L &= -[\gamma X^{-1} C_2^T + B_{12}] \\ Z &= [X(B_1 B_1^T + L B_{12}^T) + (C_1^T C_1 + C_{12}^T F)Y]/\gamma \end{aligned}$$

and $M$ & $N$ are chosen as follows. A constant $X$ and parameter-varying $Y(\bar{\rho})$ admit the controller defined by choosing

$$M = I - YX, \quad N = I$$

Similarly, a parameter-varying $X(\bar{\rho})$ and constant $Y$ admit the controller defined by choosing

$$M = Y^{-1} - X, \quad N = Y$$

Holding *both* $X$ and $Y$ constant recovers the conservative SQLF controller,[6,8] which allows for arbitrarily fast parameter variation.

## 2.3 Computing Solutions

The infinite-dimensionality of the inequalities in Theorem (2.6) demands approximate methods of solution for the sake of practical computation. One such method follows:[16,17]

Pick scalar basis functions $\{f_i \in \mathcal{C}^1(\mathbf{R}^{\bar{s}}, \mathbf{R})\}_{i=1}^{N_X}$ and $\{g_j \in \mathcal{C}^1(\mathbf{R}^{\bar{s}}, \mathbf{R})\}_{j=1}^{N_Y}$, and search over those $X(\bar{\rho})$'s and $Y(\bar{\rho})$'s that are linear combinations

$$X(\bar{\rho}) = \sum_{i=1}^{N_X} f_i(\bar{\rho}) X_i, \quad Y(\bar{\rho}) = \sum_{j=1}^{N_Y} g_j(\bar{\rho}) Y_i$$

using constant matrices $\{X_i \in \mathcal{S}^{n\times n}\}_{i=1}^{N_X}$ and $\{Y_j \in \mathcal{S}^{n\times n}\}_{j=1}^{N_Y}$. Then (2.7)-(2.9) can be rewritten as (omitting dependence in $\rho$ and $\bar{\rho}$)

$$\begin{bmatrix} \left\{ \begin{array}{c} \sum_{i=1}^{N_X} (\tilde{A}^T X_i + X_i \tilde{A}) f_j - \gamma C_2^T C_2 \\ +\sum_{k=1}^{\bar{s}} \pm\nu_k \sum_{i=1}^{N_X} \frac{\partial f_i}{\partial \bar{\rho}_k} X_i \end{array} \right\} & (\star) & (\star) \\ \sum_{i=1}^{N_X} f_i B_{11}^T X_i & -\gamma I & (\star) \\ C_1 & 0 & -\gamma I \end{bmatrix} < 0 \tag{2.10}$$

$$\begin{bmatrix} \left\{ \begin{array}{c} \sum_{j=1}^{N_Y} (\hat{A} Y_j + Y_j \hat{A}^T) g_j - \gamma B_2 B_2^T \\ -\sum_{k=1}^{\bar{s}} \pm\nu_k \sum_{j=1}^{N_Y} \frac{\partial g_j}{\partial \bar{\rho}_k} Y_j \end{array} \right\} & (\star) & (\star) \\ \sum_{j=1}^{N_Y} g_j C_{11} Y_j & -\gamma I & (\star) \\ B_1^T & 0 & -\gamma I \end{bmatrix} < 0 \tag{2.11}$$

$$\begin{bmatrix} \sum_{i=1}^{N_X} f_i X_i & I \\ I & \sum_{j=1}^{N_Y} g_j Y_j \end{bmatrix} > 0 \tag{2.12}$$

where the shorthand $(\star)$ denotes the implied transposes. The problem thus consists of finding real, symmetric matrices $\{X_i\}_{i=1}^{N_X}$ and $\{Y_j\}_{j=1}^{N_Y}$ that satisfy the above inequalities for all $\rho \in \mathcal{P}$, still an infinite-dimensional problem.

Now approximate the parameter set $\mathcal{P}$ by a grid of $L$ points $\{\rho_k \in \mathbf{R}^s\}_{k=1}^{L}$, defining $\{\bar{\rho}_k \in \mathbf{R}^{\bar{s}}\}_{k=1}^{L}$ accordingly, and solve the inequalities (2.10)-(2.12) at these grid points. The conditions represent up to $L(2^{\bar{s}+1} + 1)$ LMIs in the $N_X + N_Y$ matrix variables $\{\{X_i\}_{i=1}^{N_X}, \{Y_j\}_{j=1}^{N_Y}\}$. Note that minimizing $\gamma$ is a convex optimization problem, since the inequalities are affine in $\gamma$ as well.

# 3 Robust LPV Systems

This section derives a simple robustness test and proposes an *ad hoc* method for making the preceding control synthesis robust. Consider the analysis of an LPV system

$$\begin{bmatrix} z \\ e \end{bmatrix} = G(s,\rho)\begin{bmatrix} w \\ d \end{bmatrix}$$

put in feedback with a block-diagonal, memoryless linear time-varying (uncertainty) operator

$$w(t) = \Delta(t)z(t)$$

where $w, z \in \mathbf{R}^{n_z}$, $d \in \mathbf{R}^{n_d}$, and $e \in \mathbf{R}^{n_e}$. Assume that the block structure of $\Delta$ is consistent with a set $\boldsymbol{\Delta} \subset \mathcal{S}^{n_z \times n_z}$. Define a corresponding set of scalings

$$\mathbf{S}_{\boldsymbol{\Delta}} := \{S \in \mathcal{S}^{n_z \times n_z} : \Delta S = S\Delta \text{ for all } \Delta \in \boldsymbol{\Delta}\}$$

An elementary small-gain argument establishes the following sufficient conditions for robustness to (arbitrarily quickly) time-varying uncertainty.

**Proposition 3.1** Suppose there exists a continuous matrix function $S : \mathbf{R}^s \to \mathbf{S}_{\boldsymbol{\Delta}}$ such that for any allowable parameter trajectory $\rho : \mathbf{R} \to \mathbf{R}^s$ the resulting LTV system

$$G_S(s,\rho) = \begin{bmatrix} S(\rho) & 0 \\ 0 & I_{n_e} \end{bmatrix} G(s,\rho) \begin{bmatrix} S^{-1}(\rho) & 0 \\ 0 & I_{n_d} \end{bmatrix}$$

is exponentially stable and $||G_S(s,\rho)||_{i2} \leq \gamma$ (given zero initial conditions). Then for any allowable parameter trajectory $\rho$ and any $\Delta$ satisfying

$$\bar{\sigma}(\Delta(t)) \leq 1/\gamma$$

the LTV system $F_u(G(s,\rho),\Delta)$ also is exponentially stable and $||F_u(G(s,\rho),\Delta)||_{i2} \leq \gamma$ for zero initial conditions.

This suggests that one can design *robust* LPV controllers by alternating (in obvious fashion) between controller synthesis and computation of parameter-varying scaling matrices $S(\rho)$, as in *D-K* iteration. The design example in this paper uses an *ad hoc* choice of $S(\rho)$: "*zero*th-order" fits to frequency-dependent *D*-scales that are obtained at each "frozen" grid point by applying the appropriate $\mu$-Tools commands for closed-loop analysis. There is also a more rigorous iterative approach[2] that will not be addressed in this paper.

# 4 A Design Example

This section presents an application of LPV synthesis to the pitch rate control of the F-16 VISTA over a specified flight envelope; a similar problem has been addressed[15] using a small-gain method. The physical plant and certain design weights are parameter-varying, and a *D-K* iteration-like process is employed to enhance robust performance.

## 4.1 Plant Modelling

This design uses the standard short-period equations of motion[9]

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_\alpha(\rho) & 1 \\ M_\alpha(\rho) & M_q(\rho) \end{bmatrix}\begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} Z_{\delta_e}(\rho) \\ M_{\delta_e}(\rho) \end{bmatrix}\delta_e \tag{4.1}$$

$$\rho = \bar{\rho} = (M,h)$$

ignoring the aerodynamic effects of the trailing edge flaps. Only the longitudinal dynamics of the aircraft are considered; the roll, yaw, and sideslip angles are assumed to be zero. In (4.1) the states $(\alpha, q)$, input $\delta_e$, and parameters $(M,h)$ respectively denote angle-of-attack & pitch rate, elevator deflection, and Mach number & altitude.

The Flight Dynamics Directorate of the Wright Laboratory uses a high-fidelity, six degree-of-freedom, nonlinear model to simulate the F-16 VISTA.[1] This simulation model includes accurate descriptions of the propulsion system, actuators, sensors, disturbances, payload, atmosphere, rigid-body equations of motion, and aerodynamics for a wide range of Mach numbers, altitudes, and angles of attack.

The LPV short-period model's state space data (the dimensional coefficients $Z_\alpha$, $M_\alpha$, $M_q$, $Z_{\delta_e}$, and $M_{\delta_e}$) were obtained by trimming and linearizing the nonlinear model at level flight for the flight conditions marked in Table 1. The design region $\mathcal{P}$ was chosen accordingly, and these 21 grid points were used for the controller synthesis.

| $h\backslash M$ | 0.35 | 0.45 | 0.55 | 0.65 | 0.75 | 0.85 |
|---|---|---|---|---|---|---|
| 25000 ft | | | X | X | X | X |
| 15000 ft | | X | X | X | X | X |
| 5000 ft | X | X | X | X | X | X |
| 1000 ft | X | X | X | X | X | X |

Table 1: Grid points used for modeling & synthesis

Data on the excess thrust and rate-of-climb of the

F-16 VISTA suggest the bounds

$$\begin{aligned}\nu_1(\rho) &= 2\left(\frac{|\dot{V}_T|_{max}}{a(h)} + M^2\left|\frac{da(h)}{dh}\right|\right) &\geq |\dot{M}(t)|\\ \nu_2(\rho) &= a(h)M &\geq |\dot{h}(t)|\end{aligned}$$

on the parameters' rates of variation, where $a(h)$ denotes the speed of sound as a function of altitude and $V_T = a(h)M$ denotes the aircraft's true velocity. Note that these bounds are conservative; achieving them would require vertical flight, for example.

## 4.2 Problem Setup

The objective here is to design for the F-16 VISTA a pitch-rate controller that provides robust command tracking with predicted Level 1 handling qualities.[11] Time-domain specifications for pitch-rate response are illustrated in Fig. 1 and listed in Table 2. Note that the "rise-time" parameter $\Delta t$ varies with the true velocity $V_T$ (in ft/sec).

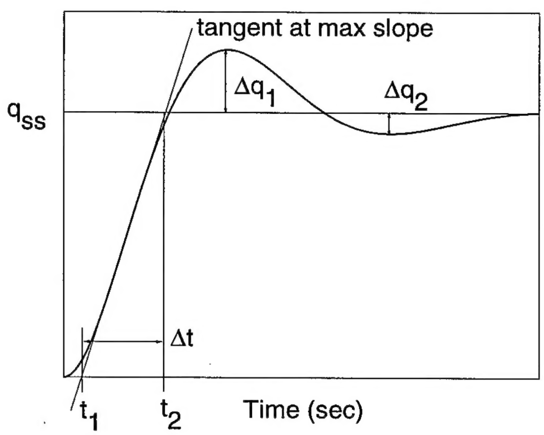


Figure 1: Pitch rate handling qualities specifications

| Parameter | Level 1 | Level 2 |
|---|---|---|
| max $t_1$ (sec) | 0.12 | 0.17 |
| max $\Delta q_2/\Delta q_1$ | 0.30 | 0.60 |
| max $\Delta t$ (sec) | $500/V_T$ | $1600/V_T$ |
| min $\Delta t$ (sec) | $9.0/V_T$ | $3.2/V_T$ |

Table 2: Pitch rate handling qualities specifications

This design uses the model-matching control structure shown in Fig. 2. The second-order reference model

$$G_{ref}(s) = \frac{\omega_n^2(Ts+1)}{s^2 + 2\zeta\omega_n s + \omega_n^2},$$

$$\omega = 4 \text{ rad/s},\ \zeta = 0.6,\ 1/T = 10 \text{ rad/s}$$

meets Level 1 flying qualities over the entire design envelope. The first-order, parameter-varying command and performance weights

$$\begin{aligned}W_r(s,\rho) &= \frac{s+100}{100}\frac{10}{s+10}q_{max}(\rho)\\ W_p(s,\rho) &= \frac{s+80}{80}\frac{4}{s+4}\frac{1}{0.05\,q_{max}(\rho)}\end{aligned}$$

reflect a uniform 10 rad/s command bandwidth, a maximum pitch-rate command $q_{max}$ that varies across the design envelope as shown in Table 3, and steady-state tracking within 5%.

| $h \setminus M$ | 0.35 | 0.45 | 0.55 | 0.65 | 0.75 | 0.85 |
|---|---|---|---|---|---|---|
| 25000 ft | | | 13 | 17 | 20 | 22 |
| 15000 ft | | 12 | 15 | 18 | 21 | 23 |
| 5000 ft | 10 | 13 | 16 | 19 | 22 | 24 |
| 1000 ft | 11 | 14 | 17 | 20 | 23 | 25 |

Table 3: Max. pitch-rate command $q_{max}$ (deg/s)

The design plant also includes actuator dynamics, additive sensor noise, multiplicative input uncertainty, and penalties on the elevator deflection angle and rate. The first-order actuator model

$$G_a(s) = \frac{20.2}{s+20.2}$$

reflects a bandwidth of 20.2 rad/s. The control weight

$$W_\delta = \begin{bmatrix} 21 & 0 \\ 0 & 70 \end{bmatrix}$$

reflects elevator deflection angle and rate limits of 21 deg and 70 deg/s, respectively. The parameter-dependent noise weight

$$W_n(\rho) = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.03\,q_{max}(\rho) \end{bmatrix}$$

anticipates 0.5 deg of measurement error for $\alpha$ and $0.03\,q_{max}$ deg/s (about 3%) for $q$. The uncertainty weight

$$W_u = 0.1$$

represents 10% parametric and/or dynamic modeling error.

## 4.3 Design Results

Several $D$-$K$ iterations (using LPV synthesis) were performed in MATLAB on the 7th-order design plant. LMILab[10] was used to solve the controller

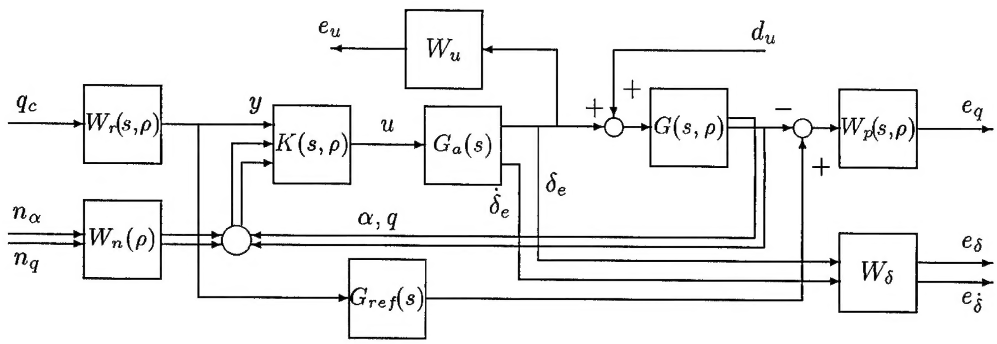


Figure 2: The design plant

synthesis LMIs, and $\mu$-Tools[5] was used for closed-loop robustness analysis. The elementary basis functions

$$[f_1(\bar{\rho})\ f_2(\bar{\rho})\ f_3(\bar{\rho})] = [g_1(\bar{\rho})\ g_2(\bar{\rho})\ g_3(\bar{\rho})] = [1\ M\ h]$$

selected according to previous experience,[2,13,16,17] were used to vary the matrices $X(\bar{\rho})$ and $Y(\bar{\rho})$ with various degrees of complexity: constant $X$ & $Y$, varying $X(\bar{\rho})$ & constant $Y$, constant $X$ & varying $Y(\bar{\rho})$, and varying $X(\bar{\rho})$ & $Y(\bar{\rho})$.

| D-K | $X, Y$ | | | $X, Y(\bar{\rho})$ | | |
|---|---|---|---|---|---|---|
| Iter. # | $\gamma$ | $\bar{\gamma}_\infty$ | $\bar{\mu}$ | $\gamma$ | $\bar{\gamma}_\infty$ | $\bar{\mu}$ |
| 1 | 2.60 | 1.90 | 1.79 | 1.49 | 1.14 | 1.13 |
| 2 | 2.43 | 1.75 | 1.73 | 1.22 | 1.05 | 1.04 |
| 3 | 2.42 | 1.74 | 1.72 | 1.19 | 1.04 | 1.03 |
| 4 | 2.42 | 1.74 | 1.72 | 1.18 | 1.04 | 1.03 |

Table 4: Closed-loop performance levels

The closed-loop time-varying performance level $\gamma$ achieved using Theorem 2.6 is shown in Table 4; constraining the rate of variation of $M$ an $h$ clearly offers a significant improvement in performance. Also included are the maximum "frozen-point" (i.e. gain-scheduled) $\mathcal{H}_\infty$ norms

$$\bar{\gamma}_\infty = \max_{\rho \in \mathcal{P}} \|G_{clp}(s,\rho)\|_\infty$$

and structured singular values (cf. Figure 3)

$$\bar{\mu} = \max_{\rho \in \mathcal{P}} \sup_{\omega \in \mathbf{R}} \mu[G_{clp}(j\omega,\rho)]$$

obtained via pointwise $\mathcal{H}_\infty$ and $\mu$-analysis of the closed-loop systems. These indicate the controller's robustness at constant flight conditions. The two designs with varying $X(\bar{\rho})$ offer no appreciable improvement over the corresponding constant-$X$ designs, so they have been omitted.

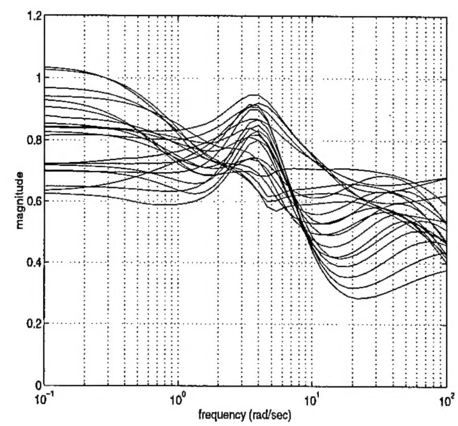


Figure 3: Closed-loop $\mu(j\omega)$ after 4th iteration

### 4.4 Nonlinear Simulation

Figure 4 shows the results of a high-fidelity, parameter-varying, nonlinear simulation of the closed-loop step response, which demonstrates predicted Level 1 flying qualities. The aircraft is initially perturbed from trimmed, level flight at $M = 0.75$ and $h = 5000$ ft. The response of the constant-$X, Y$ controller is also shown, for comparison.

## 5 Conclusions

Recent results in parameter-dependent control of linear parameter-varying systems are applied to the problem of robust, gain-scheduled pitch rate control for the F-16 VISTA. Using parameter-dependent Lyapunov functions, *a priori* bounds on the parameters' rates of variation, LMI-based convex optimization, and parameter-varying design weights, this method achieves an induced-$\mathcal{L}_2$-norm performance objective while predicting Level 1 flying qual-

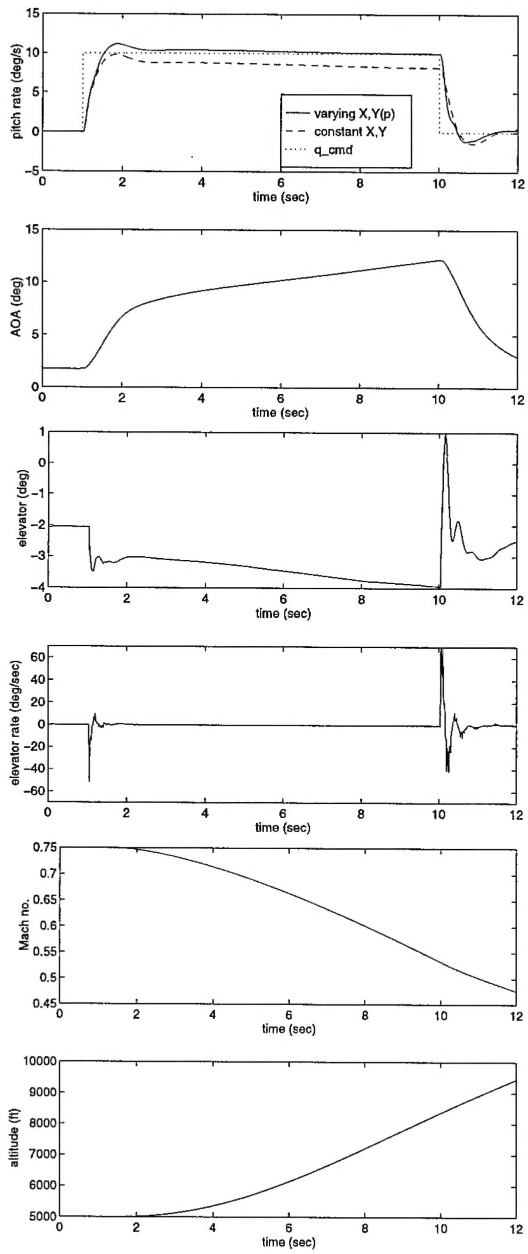


Figure 4: Parameter-varying nonlinear simulation

ities throughout the design envelope. Straightforward $D$-$K$ iteration with "constant $D$-scales" provides robustness to time-varying uncertainty. Ongoing research includes expanding the parameter set (to encompass the full flight envelope) and including a parameter-varying reference model (while maintaining Level 1 flying qualities), and incorporating the robustness scales into the synthesis LMIs.

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